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Comment on "Three-Dimensional Ascent Trajectory Optimization for Stratospheric Airship Platforms in the Jet Stream"

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Technical Comment

Comment on "Three-Dimensional Ascent Trajectory Optimization for Stratospheric Airship Platforms in the Jet Stream"

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I. Introduction

LEE and Bang [1] have recently analyzed optimal trajectories of an airship in the jet stream using a nonlinear point mass model developed in the relative wind frame. Using a point mass model for the elongated airship implies that the airship's yaw with respect to the relative wind frame is always zero; i.e., the side slip angle is zero. This is also demonstrated by the absence of side slip in the aerodynamic model. For the analysis in [1] a point mass model is adequate for analysis because the relative heading ψ, flight path angle γ, and bank angle ϕ are slowly varying such that the rotational dynamics can safely be ignored. Unfortunately, in forming the point mass model, the authors improperly consider the contribution from the added mass of the airship. In general, the added mass should be treated as a tensor in formation of the dynamics [2,3]. In [1] the tensor properties of added mass are ignored and diagonal elements of the added mass matrix are added together along with the actual mass to form a scalar total mass m_T. In addition, the added mass contribution is considered proportional to the inertial velocity of the airship rather than the relative airspeed.

II. Analysis

Development of the force from added mass begins using the same three coordinate frames as [1]: an Earth-fixed inertial frame (I frame) $Ox_iy_iz_i$, a local-level frame (h frame) $Ox_hy_hz_h$, and a relative wind frame (w frame) $Ox_wy_wz_w$. The wind and local-level frames are related by the transformation matrix C_h^w. The inertial velocity $\boldsymbol{V}_I$ is the combination of the relative flight velocity V and wind $\boldsymbol{W}_I$ and is expressed as

$$\boldsymbol{V}_I = \boldsymbol{V} + C_h^w \boldsymbol{W}_I \tag{1}$$

where

$$\boldsymbol{V} = V\boldsymbol{i}_w, \qquad \boldsymbol{W}_I = w_N\boldsymbol{i}_h + w_E\boldsymbol{j}_h \tag{2}$$

A fourth coordinate frame, the airship body frame (b frame) $Ox_by_bz_b$, must be considered to establish the relationship between the airship body and wind frame. The body frame is aligned with the airship hull with the transformation from the wind to the body frame given by

$$C_w^b = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{3}$$

where α is the hull angle of attack and considered a control variable. The definition of α is consistent with the proposed models for propeller thrust, lift, and drag in Eqs. (8) and (10) of [1]. Relative flight speed in the body frame can then be written as

$$\boldsymbol{V}_b = u\boldsymbol{i}_b + v\boldsymbol{j}_b + w\boldsymbol{k}_b = C_w^b \cdot V\boldsymbol{i}_w \tag{4}$$

The added mass force on the airship hull from acceleration of the surrounding fluid can be found by examining the fluid's kinetic energy. Following the derivation in [4], the added mass force for a body with three orthogonal planes of symmetry can be expressed compactly in the body's coordinate system using the added mass matrix $\boldsymbol{M}_a$:

$$\boldsymbol{F}_{\mathrm{AM}} = -\boldsymbol{M}_a \left.\frac{d\boldsymbol{V}_b}{dt}\right|_b - \boldsymbol{\omega}_b \times \boldsymbol{M}_a\boldsymbol{V}_b \tag{5}$$

The added mass matrix is defined as

$$\boldsymbol{M}_a = \begin{bmatrix} m_{ax} & 0 & 0 \\ 0 & m_{ay} & 0 \\ 0 & 0 & m_{az} \end{bmatrix} \tag{6}$$

where m_{ax}, m_{ay}, and m_{az} are the same added mass elements discussed in [1]. For an airship with the hull being approximately a body of revolution, it can further be assumed that $m_{ay} \cong m_{az}$. The angular velocity of the airship body with respect to the Earth-fixed frame appearing in Eq. (5) is defined as

$$\boldsymbol{\omega}_b = \dot{\alpha}\boldsymbol{j}_w + \boldsymbol{\omega}_w \tag{7}$$

where $\boldsymbol{\omega}_w$, the angular velocity of the wind frame with respect to the Earth-fixed frame, is

$$\boldsymbol{\omega}_w = p_w\boldsymbol{i}_w + q_w\boldsymbol{j}_w + r_w\boldsymbol{k}_w \tag{8}$$

Dynamic equations of motion are derived in the wind frame; therefore, it is convenient to also express the force from added mass (5) in the wind frame:

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$$
\boldsymbol{F}_{\mathrm{AM}} = -(C^b_w)^T \boldsymbol{M}_a C^b_w \begin{bmatrix} \dot{V} \\ 0 \\ \dot{\alpha} V \end{bmatrix} - \omega_b \times (C^b_w)^T \boldsymbol{M}_a C^b_w \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} \tag{9}
$$

The added mass force on the hull can be written in compact form in terms of the state derivatives $\dot{V}$, $\dot{\gamma}$, and $\dot{\psi}$ and the control variables α and ϕ by defining

$$
(C^b_w)^T \boldsymbol{M}_a C^b_w = \begin{bmatrix} m_1 & 0 & m_2 \\ 0 & m_{ay} & 0 \\ m_2 & 0 & m_1 \end{bmatrix} \tag{10}
$$

where

$$
m_1 = m_{ax}\cos^2\alpha + m_{ay}\sin^2\alpha, \qquad m_2 = \sin\alpha\cos\alpha(m_{ay} - m_{ax}) \tag{11}
$$

and using the wind frame kinematics from Eq. (7) in [1]. The final expression for the added mass force acting on the hull is

$$
\boldsymbol{F}_{\mathrm{AM}} = -\begin{bmatrix} m_1 & m_2 V\cos\phi & m_2 V\sin\phi\cos\gamma \\ 0 & -m_1 V\sin\phi & (m_2\sin\gamma + m_1\cos\phi\cos\gamma)V \\ m_2 & -m_1 V\cos\phi & -m_1 V\sin\phi\cos\gamma \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} 2m_2 V\dot{\alpha} \\ -m_2 V\dot{\phi} \\ 0 \end{bmatrix} \tag{12}
$$

Dynamic equations of motion for the airship point mass model are formed using Newton's second law. The force equilibrium is expressed as

$$
\boldsymbol{F} + \boldsymbol{F}_{\mathrm{AM}} = m \left.\frac{dV_I}{dt}\right|_I \tag{13}
$$

where the total external force $\boldsymbol{F}$ has contributions from buoyancy B, thrust T, lift L, and drag D as outlined in Eq. (8) of [1]. A comparison of the dynamic equations found using the added mass force in Eq. (12) with the formulation in [1] is facilitated by considering the case when α is small ($\sin\alpha$ is small compared to $\cos\alpha$) so that $m_1 \cong m_{ax}$ and $m_2 \cong 0$. The resulting dynamic equations found by combining Eqs. (12) and (13) then solving for the state derivatives are

$$
\begin{aligned}
\dot{V} &= \frac{(T\cos\alpha - D) - (mg - B)\sin\gamma}{m + m_{ax}} - \frac{m}{m + m_{ax}}\dot{w}_{wx} \\
\dot{\gamma} &= \frac{(T\sin\alpha + L)\cos\phi - (mg - B)\cos\gamma}{(m + m_{ax})V} \\
&\quad + \frac{m(\dot{w}_{wz}\cos\phi + \dot{w}_{wy}\sin\phi)}{(m + m_{ax})V} \\
\dot{\psi} &= \frac{(T\sin\alpha + L)\sin\phi}{(m + m_{ax})V\cos\gamma} + \frac{m(\dot{w}_{wz}\sin\phi - \dot{w}_{wy}\cos\phi)}{(m + m_{ax})V\cos\gamma}
\end{aligned} \tag{14}
$$

with $\dot{w}_{wx}$, $\dot{w}_{wx}$, and $\dot{w}_{wx}$ defined in [1]. Comparing Eq. (14) to the dynamic equations proposed in [1] two substantial differences appear. First, the total mass $m_T = m + m_{ax} + m_{ay} + m_{az}$ in [1] is replaced by $m + m_{ax}$. Because m_{ay} and m_{az} are an order of magnitude larger than both m and m_{ax}, the total mass m_T used is an order of magnitude too large. The second difference is that the wind components in Eq. (14) are multiplied by a factor $m/(m + m_{ax})$ which will be significantly less than one because both m and m_{ax} are on the same order of magnitude. When α is not small, m_2 in Eq. (12) cannot be neglected. The result is coupling between the velocity and angle equations in Eq. (14) where L, D, $\dot{w}_{wx}$, $\dot{w}_{wx}$, and $\dot{w}_{wx}$ will appear in all three dynamic equations. Because m_{ay} is an order of magnitude larger than m_{ax}, even a relatively small α of 7 deg may result in m_2 being as large as m_1.

III. Conclusions

The combination of all three diagonal elements of the added mass matrix with the actual airship mass results in a severe overestimation of the added mass's effect on the final dynamic equations in [1]. In addition, by treating the added mass contribution as proportional to the inertial velocity rather than airspeed of the airship hull, the wind's effect on the dynamic equations was also overestimated. The changes to the point mass dynamics do not alter the optimization method proposed in [1]; however, they may result in different optimal trajectories for the cases presented.

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Reply by the Authors to N. Slegers et al.

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THE authors thank N. Slegers and A. X. Brown for their comments and appreciate the opportunity given by the Associate Editor to respond to those comments. The authors carefully reviewed their paper in [1] according to the Technical Comments made by Slegers and Brown in [2]. We agree with the Comments and found that there was improper consideration of the

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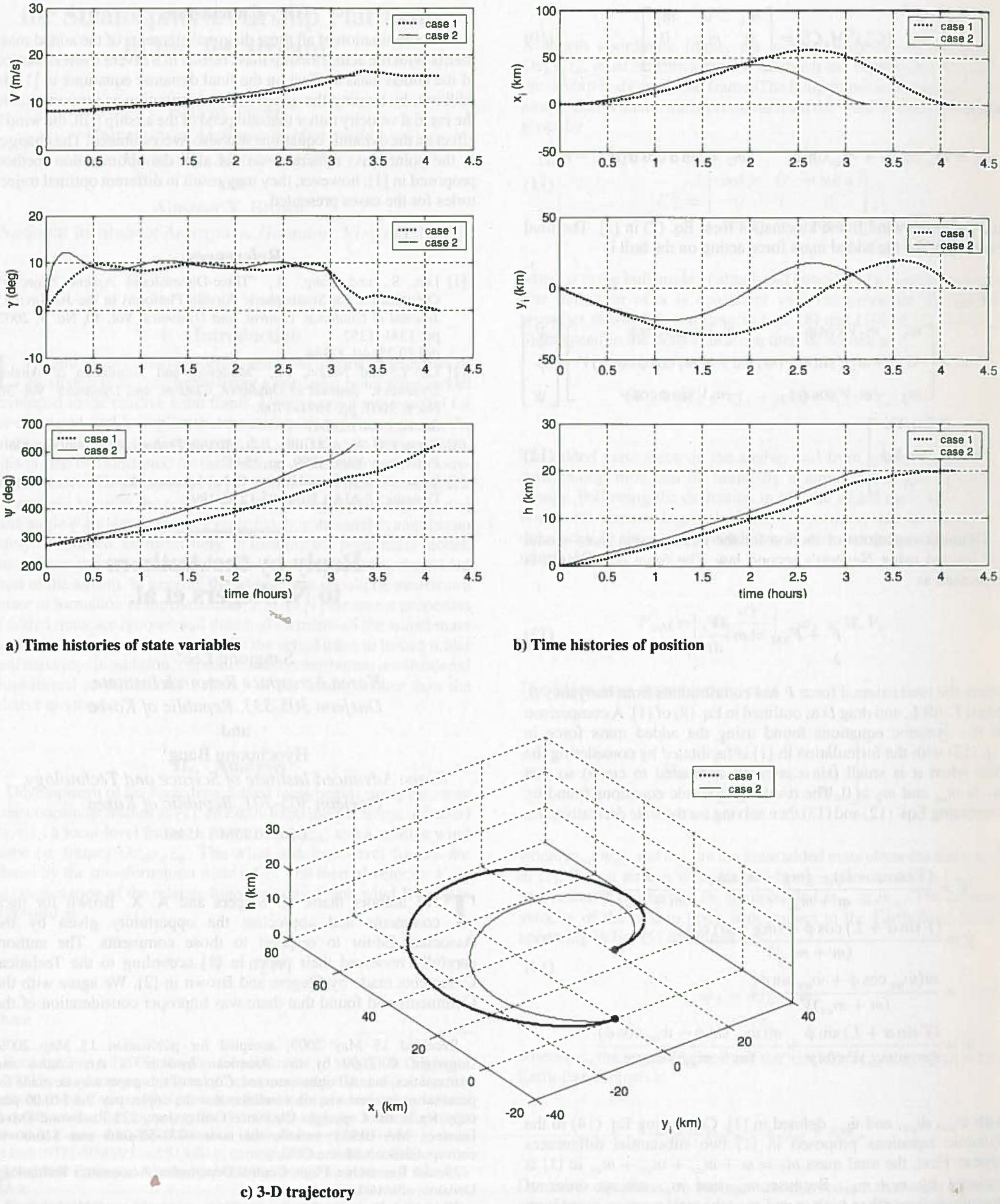
added mass term in the point mass model of our previous paper. The added mass term has been modified to generate trajectories in consideration of Eq. (14) in the Comments. Figure 1 shows minimum-time flight trajectories without jet stream and Fig. 2 presents minimum-time trajectories under jet stream condition. Cases 1 and 2 show the original and modified results, respectively. As shown in both cases, the terminal time and boundary of trajectories are decreased because the modified added mass and wind terms are accounted for the new optimization. The new added mass term is smaller than the original mass used in [1], which results in reduced maneuver time with increased speed.

In addition, the trajectories with correct added mass term sufficiently satisfy all terminal and path constraints, and their dynamic responses exhibit rather similar characteristics in comparison with

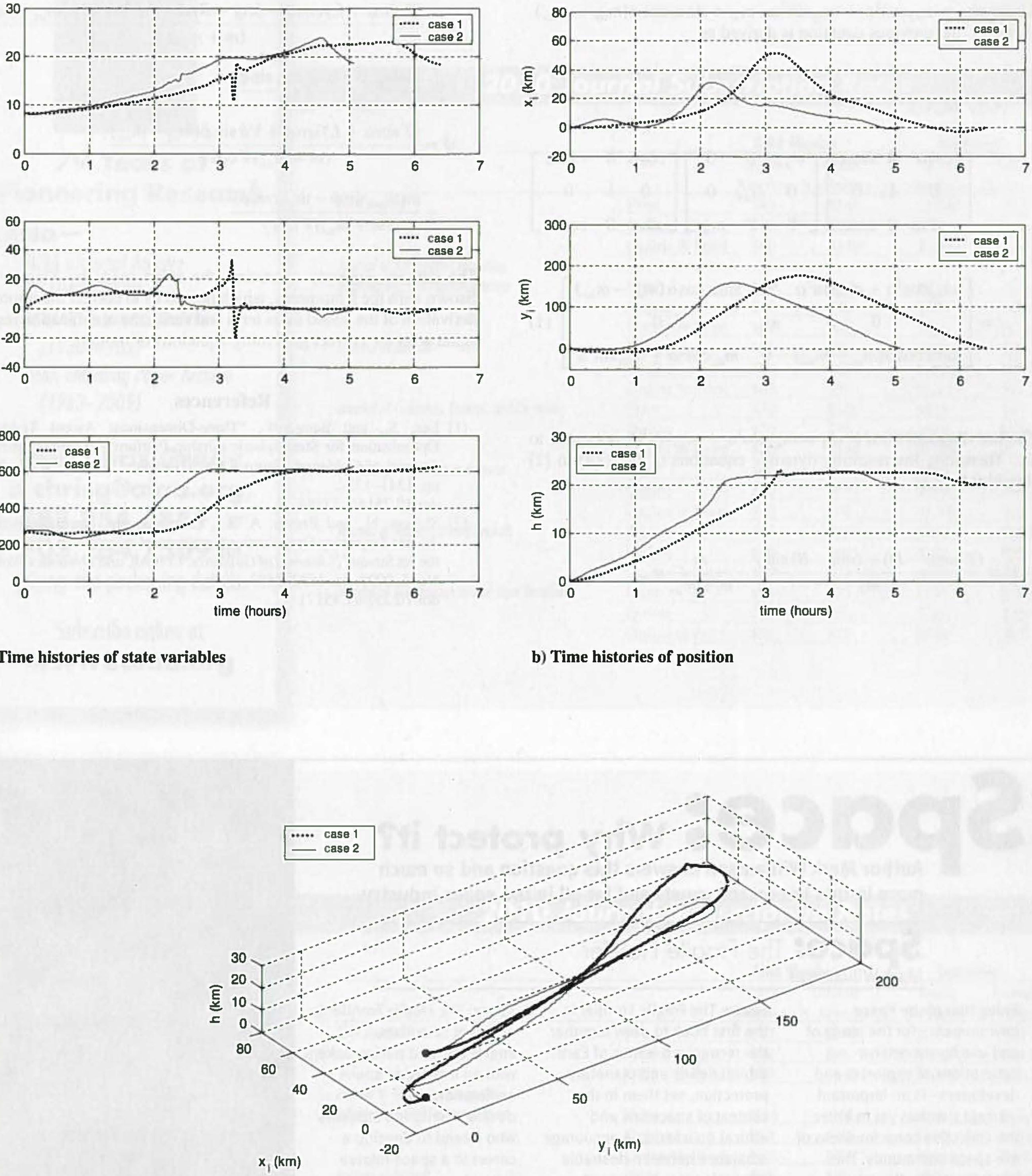


a) Time histories of state variables

b) Time histories of position

c) 3-D trajectory

Fig. 1 Comparison of the original and updated results (minimum time without jet stream).



a) Time histories of state variables

b) Time histories of position

c) 3-D trajectory

Fig. 2 Comparison of the original and updated results (minimum time with jet stream).

the original and updated trajectories. Obviously, the error in the added mass term leads to considerable difference in final 3-dimensional trajectory as Figs. 1c and 2c. However, it could be carefully said that the original definition of the problem and objectives of the work in [1] with optimization approach presented in detail still provide some useful information on optimized airship trajectory generation by considering realistic constraints.

With our best understanding, the authors find out some incorrect formulations in the Comments. In Eq. (10) of the Comments [2], it is written as

$$(\mathbf{C}_w^b)^T \mathbf{M}_a \mathbf{C}_w^b = \begin{bmatrix} m_1 & 0 & m_2 \\ 0 & m_{ay} & 0 \\ m_2 & 0 & m_1 \end{bmatrix}$$

where $m_1 = m_{ax}\cos^2\alpha + m_{ay}\sin^2\alpha$, $m_2 = \sin\alpha\cos\alpha(m_{ay} - m_{ax})$. However, the previous equation is derived as

$$(\mathbf{C}_w^b)^T \mathbf{M}_a \mathbf{C}_w^b$$

$$= \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} m_{ax} & 0 & 0 \\ 0 & m_{ay} & 0 \\ 0 & 0 & m_{az} \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} m_{ax}\cos^2\alpha + m_{ay}\sin^2\alpha & 0 & \sin\alpha\cos\alpha(m_{ay} - m_{ax}) \\ 0 & m_{ay} & 0 \\ \sin\alpha\cos\alpha(m_{ay} - m_{ax}) & 0 & m_{az}\cos^2\alpha + m_{ax}\sin^2\alpha \end{bmatrix} \quad (1)$$

The last diagonal term is $m_3 = m_{az}\cos^2\alpha + m_{ax}\sin^2\alpha$, not equal to m_1. Therefore, the resulting dynamic equations of Eq. (14) in [2] should follow as

$$\dot{V} = \frac{(T\cos\alpha - D) - (mg - B)\sin\gamma}{m_T} - \frac{m}{m + m_{ax}}\dot{w}_{wx}$$

$$\dot{\gamma} = \frac{(T\sin\alpha + L)\cos\phi - (mg - B)\cos\gamma + V\dot{\alpha}\sin\phi(m_{az} - m_{ax})}{(m + m_{ax})V} + \frac{m(\dot{w}_{wz}\cos\phi + \dot{w}_{wy}\sin\phi)}{(m + m_{ax})V}$$

$$\dot{\psi} = \frac{(T\sin\alpha + L)\sin\phi + V\dot{\alpha}\sin\phi(m_{az} - m_{ax})}{(m + m_{ax})V\cos\gamma} + \frac{m(\dot{w}_{wz}\sin\phi - \dot{w}_{wy}\cos\phi)}{(m + m_{ax})V\cos\gamma} \quad (2)$$

The authors sincerely appreciate the effort made by Slegers and Brown with the Comments, which helped us to correct the improper derivation of the added mass term and verify the optimization results again with the correct governing equations of motion.

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