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PRESERVICE TEACHERS' TEMPERATURE STORIES FOR INTEGER ADDITION AND SUBTRACTION

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Ninety-eight elementary and middle school preservice teachers posed eight stories for integer addition and subtraction number sentences. Stories that were posed about temperature were analysed using a modified Marthe's (1979) framework for integer problem types. This framework was modified based on the stories provided by the preservice teachers. This paper reports on the problem types utilized by the preservice teachers. Results highlight that preservice teachers do not frequently use some problem types. Also, results may indicate that some number sentence types (e.g., -23 – -5=☐) support different problem types (e.g., State-State-Translation).

INTRODUCTION

Negative numbers and operations about them are notoriously challenging. Piaget (1948) reflected on this noting that, "Everyone knows the difficulty that secondary students [and even university students!] have in understanding the algebraic rules of signs – 'minus times minus equals plus'" (p. 104). This is further complicated when our students are asked to reason conceptually about integers or apply integers to various contexts.

BACKGROUND LITERATURE

Research focusing on student thinking about integers and the ways young children reason has gained recent momentum in the field (e.g., Bofferding, 2014; Bishop et al., 2014). Within this increased research about integers, a subset has focused on understanding the role of context within the realm of student thinking (Stephan & Akuyz, 2012; Whitacre et al., 2014). Yet, research situated in making sense of how preservice teachers (PSTs) reason about and use integers has mainly focused on their solution strategies to integers arithmetic problems (Bofferding & Richardson, 2013; Chrysostomou & Mousoulides, 2010). However, making sense of integers within contexts is important for PSTs to develop, as they will be teaching this topic to their students in the future.

Both children and PSTs have difficulties with creating contexts for integer operations (Kilhman, 2009; Mukhopadhyay, 1997; Wessman-Enzinger & Mooney, 2014). Mukhopadhyay (1997) asked 32 students in grades 5, 6, and 7 to solve problems involving negative integers and tell a story that matched the equations. Four case studies were provided that demonstrated that students struggled to generate stories. She hypothesized that this was attributed to the various mental models the students were possibly employing. Similarly, Kilhman (2009) asked 99 PSTs to solve and describe their thinking for number sentences (e.g., -8 – -3= ☐). Of the 99 PSTs, Kilhman found that only 23 utilized a model or context to explain the mathematics and did so with either number lines and/or temperature to explain their reasoning.

Research with children has found that stories they pose for integer addition and subtraction can be classified in to the Conceptual Models for Integers of Addition and Subtraction (CMIAS), or ways of mathematically reasoning about and using the integers (Wessman-Enzinger & Mooney, 2014). Temperature was found to be a useful context for developing two CMIAS, *Translation* and *Relativity*. For example, *translation* concepts surface when a context suggests increasing or decreasing a temperature. *Relativity* conceptions develop because the temperature scale itself is a relative scale with an arbitrarily, although intentionally, selected zero.

THEORETICAL PERSPECTIVE

For informing the translation conceptual model, it is important to understand contextual problem types that may support those ways of thinking. Marthe (1979) , in the first paper published about negative integers in PME proceedings, classified different problem types for additive structures for integers. The first category was S_iTS_f, where the initial state (S_i) is translated (T) to the final state (S_f). Marthe then described that S_i, T, or S_f could represent the unknowns in any given problem. A second category was $T_1T_2T_3$. He described T_1, T_2, and T_3 as "transformations," although they can also be described as linear translations. From this problem type, Marthe described that there are three subsequent problems that can be posed, where T_1, T_2, or T_3 are unknowns having differing magnitudes and signs. For example, $T_1T_2T_3$ with T_2 unknown, T_1 and T_3 with opposite signs, and $|T_1| < |T_3|$, could be contextualized as, "A car makes an initial journey of 20 km upstream. Then it makes a second journey. If it had made only one journey from its starting-point to its destination, it would have made a journey of 25 km downstream. Describe the second journey" (Marthe, 1979, p. 156). Marthe also included a category, SSS, which is composed of all states and no translations.

Although Marthe did not provide examples for temperature, his problem types are applicable to the context of temperature. The dropping of temperature can be compared to travelling downstream; and, a relative position on a stream can be compared to the relative position on a temperature scale. Identifying the types of contextualized that PSTs pose can shed light into the difficulties that they may have and ways to support their learning as future mathematics educators.

Research Question

What kinds of stories about temperature do PST elementary and middle school teachers pose for integer addition and subtraction number sentences?

METHODOLOGY

Ninety-eight elementary and middle school PSTs participated in a study focusing on integer addition and subtraction while enrolled in an introductory mathematics content course. The authors, who are also professors for this course, analysed the written tasks from their students. The mathematics content course is designed to promote conceptually-oriented discourse around number and operations. To prepare PSTs to become mathematics educators, they are encouraged solve problems in multiple ways, present their own solution strategies, and understand the reasoning of others (Cobb & Yackel, 1996).

Data Collection

Data was collected across two academic semesters, Fall 2013 and Spring 2014. The PSTs were given 8 integer addition and subtraction number sentences (i.e., 16 – 4 = ☐, -17 + 12 = ☐, 18 + -13 = ☐, 8 – 20 = ☐, -2 – 3 = ☐, -14 – -20 = ☐, -6 + -9 = ☐, -23 – -5 = ☐) and asked to pose stories that they thought best matched these number sentences. This task was given to the PSTs prior to instruction on integer operations and reasoning about integers contexts in the course. Although other integer tasks were given to the PSTs, this preliminary task is the focus of this paper.

Data Analysis

The 98 PSTs posed 8 stories each for a total of 784 stories. Of these, 108 (13.8 %) were posed utilizing the context of temperature. The 108 stories about temperature constitute the unit of analysis for this study. These 108 temperature stories were organized by integer addition and subtraction problem types and examined for themes. The authors identified themes such as: the realism of context, mathematical correctness, and the consistency of the problem type to the story. Although these themes became codes, this paper reports on the themes about problem types that emerged and were guided by Marthe's (1979) macro-problem types (i.e., STS, TTT, SSS).

The authors used constant comparative methods (Glaser & Strauss, 1967) to analyse the temperature stories. After an initial pass through of the temperature stories, the authors discussed and agreed on modifying Marthe's STS problem type by extending it to *State-State-Translation* (SST) and *State-State-Distance* (SSD). Our SST is similar to Marthe's problem type STS with T unknown, but we felt that making the permutations of the letters explicit captured an imperative difference in the problem types SST and STS. For SST, the S's represent two relative numbers and T represents a translation from one relative number to another. We also wanted to make directed distance explicit, which is why we added the problem type SST and differentiated it from SSD, where the D represented distance without established direction. Although one may mathematically argue that all distance is directed, the PSTs posed stories in a way that direction was relative. We maintained Marthe's problem types TTT and SSS. After the authors agreed upon these modified problem types (i.e., STS, SST, SSD, TTT, SSS), the authors coded each of the 108 temperature stories posed by the PSTs with these codes. The authors agreed on 92 of the 108 codes or 85.2% of the time. All of the disagreements were negotiated and resolved.

RESULTS & DISCUSSION

Results are reported by problem type (i.e., STS, SST, SSD, TTT, and SSS). Though no stories were posed by the PSTs about temperature for 16 – 4 = ☐, the other seven number sentences each had ten or more stories about temperature posed for them.

STS Problems

When a story was posed with a relative temperature and a translation with the second relative temperature as the unknown, it was considered to be an STS problem. The PSTs posed STS problems the most for the number sentences -17 + 12 = ☐ and 8 – 20 = ☐ (See Table 1).

Number Sentence	STS Problem Type
-17 + 12 = ☐	21/23 (91.3%)
18 + -13 = ☐	8/14 (57.1%)
8 – 20 = ☐	11/13 (84.6%)
-2 – 3 = ☐	13/16 (81.3%)
-14 – -20 = ☐	11/16 (68.8%)
-6 + -9 = ☐	12/16 (75%)
-23 – -5 = ☐	6/10 (60%)

Table 1: Number Sentence and STS Problem Type

A few common examples of STS problems that the PSTs posed for these number sentences are shown below.

PST 31: The temperature is 8 degrees and then it goes down 20 degrees. What is the temperature now?

PST 91: In New York the temperature was -17°F in the morning. If the temperature went up 12°F, what is the temperature?

PSTs posed STS problems the least for the problem types 18 + -13 = ☐ and -23 – -5 = ☐. Some common examples of the STS problem posed for 18 + -13 = ☐ are shown below.

PST 26: During the day, the temperature was 18 degrees. By the end of the day, the temperature decreased by 13 degrees. What temperature was it by the end of the day?

PST 29: The temperature is 18 degrees and it goes up by -13 degrees. What is the temperature?

PST 18: The temperature is 18°. By tonight it will drop -13. What will the temperature be?

Although each of these examples is considered to be a STS problem type, there are notable distinctions between these stories. PST 26's story is for 18 – 13 = ☐, which is equivalent to 18 + -13 = □ Although PST 29 posed a story that is mathematically equivalent to 18 + -13, it is not realistic to talk about temperature increasing by a negative number. However, PST 18 posed a story that is not mathematically equivalent to 18 + -13 = □ and instead posed a story for 18 – -13 = □, which is also not realistic.

Number sentences like -17 + 12 = □ and 8 – 20 = □ seem to support STS problems more than number sentences like 18 + -13 = □ and -23 – -5 = □

SST and SSD Problems

When a story was posed with two give relative temperatures and the translation is unknown, it was considered to be an SST problem. Whereas, when the story was provided with two relative temperatures and a distance, without direction, it was considered to be an SSD problem. Although not mathematically correct, PST 25 posed a SST problem for the number sentence -17 + 12 = □

> PST 25: It was 12° outside Wednesday. It was 17 below zero degrees Thursday. How much had the temperature dropped since Wednesday?

In this story both the temperatures are provided, and the question provides a distinct direction by indicating Wednesday to Thursday. Interestingly, the SST problem type was utilized more for problems like -17 + 12 = □, rather than other more reasonable number sentences, like -14 – -20 = □ (see Table 2). For example, PST 74 provided an example of an SST problem that works well.

> PST 74: It is -23°F in Antarctica and it is -5 degrees in Illinois. What is the difference between Antarctica's temperature and Illinois?

The distinguishing feature of the stories posed that were consider to the SSD problem type is that no direction is provided in the stories. The most common number sentence used for the SSD problem type was -14 – -20 = □ (see Table 2). A common story for this number sentence and problem type is shown below.

> PST 4: One day in New York it is -14 degrees out. In Maine the same day it was -20 degrees. What is the difference between the two states' temperatures?

Again, the direction is not established. Thus, one could use both number sentences -14 – -20 = □ and -20 – -14 = □ to describe this story. -14 – -20 = 6 could be described as representing 6 degrees warmer in New York. Whereas, for -20 – -14 = -6, the -6 could be described as representing 6 degrees colder in Maine. The SSD problem type was used more for problem types like -14 – -20 = □ and -23 – -5 = □ This could point to evidence that these number sentences could promote use of these problem types more. Overall, many PSTs did not use either the SST or SSD problem types frequently. Instead of SST or SSD problem types for number sentences like -14 – -20 = □, PSTs would often pose stories like:

> PST 14: The temperature last night was -14°F. When I woke up, it had gone up 20°. What is the temperature right now?

Although -14 – -20 = ☐ is mathematically equivalent to -14 + 20 = ☐, the stories posed for both are not equivalently appropriate. PSTs need to be able to pose stories for -14 – -20 = ☐ that are appropriate and realistic, rather than just posing mathematically equivalent stories for -14 + 20 = ☐.

Number Sentence	SST Problem Type	SSD Problem Type
-17 + 12 = ☐	2/23 (8.7%)	0/23 (0%)
18 + -13 = ☐	1/14 (7.1%)	1/14 (7.1%)
8 – 20 = ☐	1/13 (7.7%)	1/13 (7.7%)
-2 – 3 = ☐	1/16 (6.3%)	2/16 (12.5%)
-14 – -20 = ☐	0/16 (0%)	5/16 (31.3%)
-6 + -9 = ☐	0/16 (0%)	1/16 (6.3%)
-23 – -5 = ☐	1/10 (10%)	3/10 (30%)

Table 2: Number Sentence and SST & SSD Problem Types

TTT and SSS Problems

None of the PSTs in this study posed stories for the TTT problem type presented by Marthe (1979). However, some PSTs posed stories where two relative temperatures were provided and a third relative temperature was unknown, a SSS problem type.

The SSS problem type was only used for the number sentences 18 + -13 = ☐ and -6 + -9 = ☐. The following examples are stories considered to be the SSS problem type for the number sentence -6 + -9 = ☐.

PST 85: It is -6 degrees outside at 12 pm. At 12 am another -9 degrees is added. How many degrees is it at 12 am?

PST 91: It is -6°F in Bloomington and -9°F in Chicago. What is the sum of the two temperatures?

Number Sentence	SSS Problem Type	Other
-17 + 12 = ☐	0/23 (0%)	0/23 (0%)
18 + -13 = ☐	2/14 (14.3%)	2/14 (14.3%)
8 – 20 = ☐	0/13 (0%)	0/13 (0%)
-2 – 3 = ☐	0/16 (0%)	0/16 (0%)
-14 – -20 = ☐	0/16 (0%)	0/16 (0%)
-6 + -9 = ☐	2/16 (12.5%)	1/16 (6.3%)
-23 – -5 = ☐	0/10 (0%)	0/10 (0%)

Table 3: Number Sentence and SSS & Other Problem Types

Piaget (1952) referred to temperature as an example of how he defined intensive quantities, pointing out that temperatures cannot be summed. Here, the context of temperature, the SSS problem type is not appropriate. Perhaps some of the PSTs posed these types of stories for number sentences like 18 + -13 = ☐ and -6 + -9 = ☐ because there are not many temperature contexts that are appropriate for these number sentences. It seems that the SSS problem type is not even appropriate for the Translation CMIAS. Three stories that used wind chill incorrectly were classified as other because there wasn't enough information to determine if the PSTs intended the problem to be TTT or STS. For example, a PST wrote, "It's 18 degrees outside. There is a wind chill of -13 degrees. How warm does it feel outside?"

CONCLUSION

Most of the PSTs posed temperature stories that were the STS problem type. Although it is certainly important for PSTs to be able to make sense of integer addition and subtraction with the STS problem type, it is also important for PSTs to utilize other problem types like SST, SSD, and TTT. This study also provided evidence that certain integer addition and subtraction number sentences facilitate reasoning about integers in different ways and with different problem types. Number sentences like 18 + -13= ☐ and -23 – -5= ☐ are potentially rich areas for robust discussions with PSTs given their struggles for appropriate problem types for these number sentences. For example, if it is -23 degrees in Chicago and -5 degrees in Boston, then discussions about how both -5 – -23= ☐ and -23 – -5= ☐ could fit that story would be productive. Because certain number sentences may support some problem types more than others, a variety of number sentences should be given when teaching integers to facilitate reasoning in different ways. Identifying PSTs' conceptions, like we did in this paper, is instrumental to develop rich instructional tasks and understand how to prepare them to become future mathematics educators.

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