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ALICE'S DRAWINGS FOR INTEGER ADDITION AND SUBTRACTION OPEN NUMBER SENTENCES

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Alice, a fifth grader who participated in twelve weeks of a teaching experiment on integer addition and subtraction, produced drawings as part of her strategy for solving integer addition and subtraction open number sentences. The drawings she created during the twelve weeks of the teaching experiment were analyzed and grouped into the following categories: Single Set of Objects, Double Set of Objects, Number Paths & Number Lines, and Number Sentences. These drawings provide insight into how children may directly model or count when solving integer addition and subtraction problems.

Keywords: Number Concepts and Operation; Elementary School Education; Cognition

For solving addition and subtraction problem with positive integers we know that children often use strategies that incorporate drawings that include direct modeling, counting, or derived facts (Carpenter, Fennema, Franke, Levi, & Empson, 2015). Children often use drawings paired with direct modeling or counting strategies as they begin to invent strategies for solving addition and subtractions problems. We also know that children often draw upon direct modeling strategies, which may incorporate drawings, when the number size changes. However, as a field, we know little about the drawings that children employ as they transition from using positive integers to negative integers. Bofferding (2010) demonstrated that children often use a number path when solving integer problems. Other researchers have shown that children will use a variety of ways to reason about the integers which include order-based or number line reasoning (e.g., Bishop et al., 2014). Despite the insurgence of research on the ways that student think about integers (e.g., Bofferding, 2014; Bishop et al., 2014; Wessman-Enzinger & Mooney, 2014), we need to know more about the ways that children reason about integers in relationship to they ways that children employ direct modeling or counting. One way to identify more of these direct modeling and counting strategies from children is to look at the drawings that they produce and create. Vig, Murray, Star (2014) highlight that understanding the productive aspects of models, as well as their breaking points, is an important component to integer addition and subtraction. Understanding ways that children use these drawings productively and unproductively could help provide insight into affordances and hindrances of models.

Theoretical Perspective

Word use, visual mediators, narratives, and routines are the central tenets of discourse in commognitve theory (Sfard, 2008). Although all of the tenets of commognitive theory work together synergistically, the visual mediators are the focus of this paper. Visual mediators include recognizing artifacts such as gestures or drawings as part of a students' discourse. Drawings that children produce while solving integer addition and subtraction open number sentences represent a component of their discourse that is just as important as their verbal reasoning. For students, drawings can be as communicative as their verbal expressions and investigations into their drawings for negative integers can be illuminative. During the time with the Grade 5 students, they often used drawings to help them make sense of the negative integers. This research brief highlights one of these three students, Alice, and her drawings. Specifically, this research brief addresses the research question:

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What types of drawings does Alice produce as she solves integer addition and subtraction open number sentences?

Methodology

Three Grade 5 students from a rural Midwest school participated in a 12-week teaching experiment (Steffe& Thompson, 2000) centered on integer addition and subtraction, using both contextual problems and open number sentences. The students met in both individual and group sessions during the teaching experiment and all sessions were videotaped. The students primarily solved problems in contexts during both the individual and group sessions; however, there were four individual sessions where students solved open number sentences.

Integer addition and subtraction open number sentences were solved during four individual sessions across the 12-weeks. During these sessions the open number sentences were provided on paper, with no manipulatives and only a box of markers available. The students were asked to explain their reasoning for solving the open number sentences. Alice was chosen as the participant to report on in this research brief because of the three participants Alice used drawings the most. Alice's drawings from the individual sessions with open number sentences represent the unit of analysis. A grounded theory approach (Glaser & Strauss, 1967) was utilized for categorizing the different types of Alice's drawings. Both the verbal interactions from Alice and the teacher-researcher, as well as, the process of her drawings were transcribed. Each of the drawings, paired with these descriptions and transcripts, was examined and sorted for common themes.

Results & Discussion

Single Set of Objects

Alice often drew a Single Set of Objects to solve the open number sentences. The Single Set of Objects were utilized in two different ways, by either crossing off objects or adding objects on. For crossing off a Single Set of Objects, Alice began by drawing an initial set of objects (e.g., boxes or tallies), which represented either a positive or negative integer. She then crossed some of the objects off (see Figure 1). The objects crossed off represented either the addition or subtraction of a positive or negative integer. Alice's objects that she drew included either boxes or tallies for the Single Set of Objects.

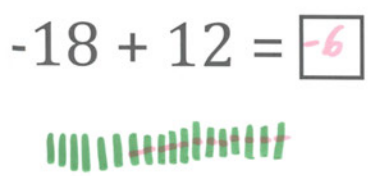

Figure 1: Single Set of Objects for Solving -18 + 12 = □

In Figure 1, Alice used 18 tallies to represent the negative integer, -18. Then, Alice crossed off 12 tallies, representing the positive integer being added.

Double Set of Objects

Other drawings that Alice produced frequently included two layers or two separated groups of objects. The drawings that included these layered or separated objects were considered a Double Set of Objects (see Figure 2). For example, Alice used the Double Set of Objects drawings with when solving □+ -4 = 13. In Figure 2, the pink tallies represent negative four and the green tallies represent 17. Alice added tallies until the leftover tallies totaled 13. Then, she counted all of the green tallies to determine the solution of 17.

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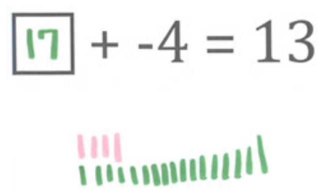

Figure 2: Double Set of Objects for Solving□ + -4 = 13

Although Alice would use objects that were layered on top of each other, Alice also represented the addition and subtraction with segregated layers. For example, in Figure 3, Alice represented the -4 with boxes and then segregated the second set of boxes, but this layer of boxes was not stacked on top of the other boxes like in Figure 2. She then added up all of the boxes to get 14. Alice described her drawing, "I did four for negative four (motions across four boxes), then I did how many I was adding a box for how many it would take to get me up to ten." Although this type of drawing is reminiscent of a Number Path (see, e.g., Bofferding, 2010; Wessman-Enzinger & Bofferding, 2014), Alice did not recognize these boxes as orderings of numbers. Instead, she described the quantities -4 and 10 without order.

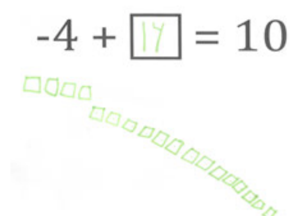

Figure 3: Double Set of Objects for Solving -4 + □= 10

Number Sentences

Alice often drew horizontal or vertical number sentences to solve the open number sentences. Sometimes the horizontal or vertical number sentences used only positive integers, while sometimes the horizontal or vertical number sentences incorporated negative integers. For example, to solve -12 – -11 = □, Alice drew a vertical number sentence involving negative integers. She vertically wrote, -11 – -12. Yet, Alice still obtained -1 as a solution. This is consistent to findings that children often incorrectly apply the commutative property when subtracting negative integers (Bofferding, 2010).

Number Path & Number Lines

Alice only drew a Number Path with negative integers once during all of the individual sessions (see Figure 4). Alice did not draw a conventional Number Line; rather, she drew a Number Path (see, e.g., Bofferding, 2010; Wessman-Enzinger & Bofferding, 2014).

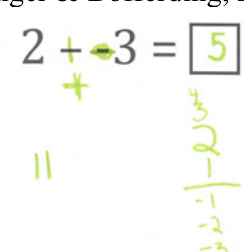

Figure 4: Number Path for Solving 2 – -3 = □

Her drawing in Figure 4 included the ordering of a Number Line and has a close relationship to the formal Number Line, yet is not a Number Line. She drew this Number Path after solving the open number sentences 2 – -3 = □ during the last individual session of the twelve weeks. To solve 2 – -3 =

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□, Alice first solved it by recalling a rule that the student developed during the group sessions, "Because it's just like the last one. You do plus (changes minus sign to plus sign) and take that off (scratches off the negative symbol of -3). And, it just be like two plus three." When asked why it worked, she drew both a Single Set of Objects (e.g., tallies) and then a Number Path to try to justify her reasoning. Although she drew a Number Path, she didn't utilize it to solve or justify her solution. In fact, Alice shared that she didn't know how to use either of her drawings (Single Set of Objects or Number Path) to explain her answer.

Conclusion

Alice used a variety of drawings that were productive for solving integer addition and subtraction number sentences. Despite the different types of drawings she drew, she did not draw very many Number Paths or any Number Lines during the individual sessions. Instead, Alice drew the quantities of objects that seem to be related to direct modeling strategies (Carpenter et al., 2015). Towards the end of the teaching experiment Alice began to utilize a Number Path, which may highlight that the development of drawing Number Lines takes extended time for some children. This may point that the development of using Number Paths and Number Lines takes significant for some students. These different types of drawings (e.g., Single Set of Objects, Double Set of Objects) provide further insight into the ways we understand student thinking about addition and subtraction with integers.

Acknowledgements

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