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A New Spin on Fair Sharing

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A New Spin on Fair Sharing

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Students often have difficulties making connections between rational number concepts and their relationships to other mathematical applications and real world situations (Johanning, 2008). Researchers have advocated that students should experience using rational numbers with multiple and varied models integrated into context (Empson & Levi, 2011). In this article, we discuss a lesson that drew upon probabilistic reasoning as a means to help students connect rational number reasoning to real world situations. Probabilistic situations act as an extension to rational numbers in that they often involve fractional models and encourage students to reason through topics, such as part to whole relationships and fractional equivalence. Even though probabilistic reasoning is often clouded with misconceptions, it involves the ability to integrate rational number reasoning into a context with discussion and justification rooted in rational number thinking (Jones et al., 1997).

The Common Core State Standards for Mathematics suggest rational number equivalence should be addressed in the third and fourth grades. While working with a fourth grade classroom, we thought probabilistic comparisons might be an ideal context to elicit students' conceptions about fairness and rational number equivalence. We wanted to draw on students' knowledge of fair sharing in relation to their probabilistic reasoning. A fair sharing problem involves a number of items that need to be shared among a given number of people or groups (Empson & Levi, 2008; Wilson et al., 2012). We wanted to see if students' understanding of fraction equivalence would translate into their understanding of probability and fairness.

Keeping these ideas in mind, we began to plan the lesson and decided to create a scenario that centered on winning a game. We generated several spinners that each represented the same chance of winning but were composed of different size pieces and also arranged in different ways (see Figure 1).

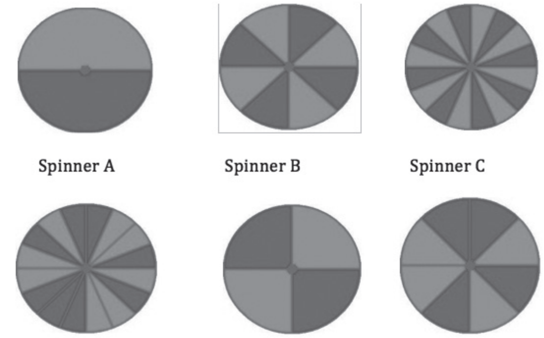


Figure 1. Spinners.

Researchers have indicated that it is important for students to see multiple representations of fractions beyond the circle model, like set models, fraction bars, area models, and number lines (Petit, Laird, & Marsden, 2010). Although we recognized that multiple models are important, we decided to focus on a singular fraction model for this lesson. We thought that one fractional model, specifically the circle model, would be best to help draw the students' attention to comparison and equivalence. We decided to make the spinners all varying representations of one-half utilizing the circle model and, depending on the results, we could explore other fractions as an extension.

Below we present this two-day lesson that aimed to introduce and elicit students' reasoning about fractional equivalence through the probabilistic concept of fairness.

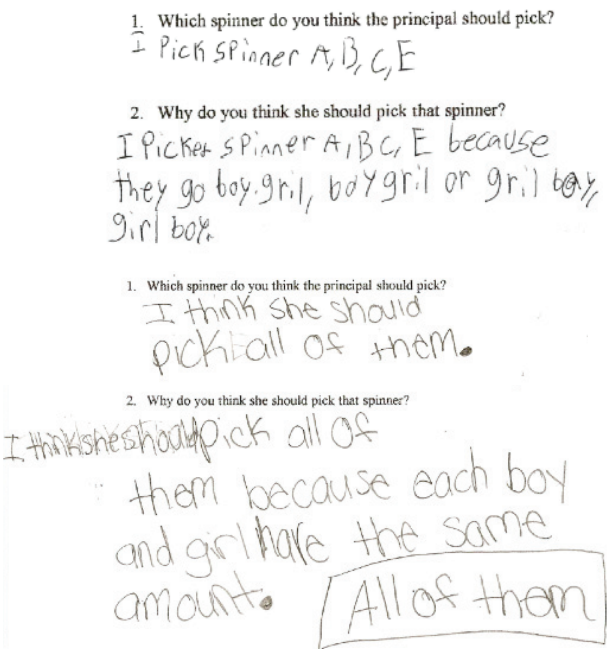
Lesson Day 1

On the first day of the lesson the students began with an introduction to the problem:

> *The boys and girls in the class are playing a game against each other. If the spinner lands on blue the girls get a point and if the spinner lands on red the boys get a point. Which spinner or spinners would you choose for the game?*

Before we gave the students the spinners to test, they were asked to explain which spinner or spinners they would choose and why. We gave them this prompt to see what initial conceptions or misconceptions they might have to help us guide the lesson. Of the student responses, half of the students picked Spinner A as the spinner they would use. This was primarily because they thought that the boys and girls had what seemed to be more area for the spinner to land on. Other students also picked A because they felt it was the best representation of equal.

Some students were concerned with the order of the sectors on the spinners. They indicated that they should use spinners that had sectors that alternated colors (i.e., Spinners B, C, and E) otherwise it wasn't fair. Only two or three students initially responded that all of the spinners would work because they recognized that the spinners each represented one half even though they were different in appearance. Examples of their work are shown below in Table 1.



1. Which spinner do you think the principal should pick?
I Pick sPinner A, B, C, E

2. Why do you think she should pick that spinner?
I Picked sPinner A, B C, E because they go boy. gril, boy gril or gril boy, girl boy.

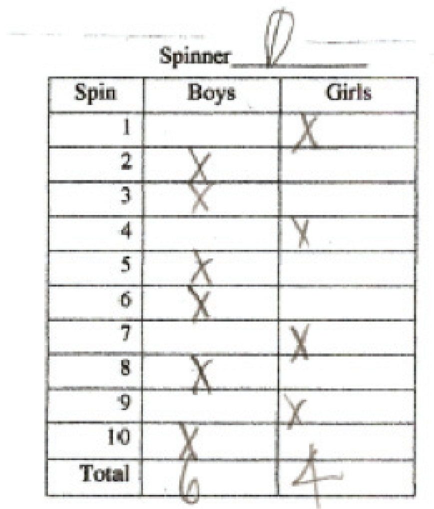
1. Which spinner do you think the principal should pick?
I think she should pick all of them.

2. Why do you think she should pick that spinner?
I think she should pick all of them because each boy and girl have the same amount. All of them

Following this reflection, we had each of the students spin each of the spinners ten times and record their findings to determine who won for each spinner (See Figures 2, 3 and 4). The students took turns spinning the spinners and exchanging them with classmates. Testing the spinners took the remainder of the time for mathematics and the lesson concluded with the students submitting their results to us.

Figure 2. Students collecting data.

Figure 3. More students collecting data.



Spinner [illegible]

Spin	Boys	Girls
1		X
2	X	
3	X	
4		X
5	X	
6	X	
7		X
8	X	
9		X
10	X	
Total	6	4

Figure 4. Student recording table.

Lesson 1 Reflection

Following the first day of the lesson, we realized that spinning the spinners only ten times was not enough. The students needed experience with spinning the spinners many times. We decided that we would compile the students' results and bring in the Law of Large Numbers to direct the students' focus to the layout of the spinners. The Law of Large Numbers states that the more times an experiment is performed the closer the results will be to the expected value. In our case, the greater the number of spins the closer the numbers would be to girls winning half of the time and boys winning half of the time. We heard several of the students mention the word fairness in the lesson, so we decided to begin the second lesson with a discussion about the fairness of the spinners. We felt that this would help the students to begin to focus on rational number equivalence.

Lesson Day 2

On the start of the second day of the lesson, the students were told that we compiled all of the spinner results so that we could see what happened if the spinners were each spun around 200 times (see Figure 5). Without showing them the results, we asked the students what they expected to see. We noted that the word "fairness" had come up in conversation several times the day before and asked the students what they thought the word fair meant. The students responded that they thought fair meant that each person would win the same amount of times. We then directed their attention to the spinners, and asked what a spinner would look like if it was fair and what results would we see from a fair spinner. Several of the students said that fair for the spinners would mean that there was a 50/50 chance of winning. When we asked the students to explain, they stated that each person should win half of the time or nearly half of the time. One of the students stated that if the spinners were fair and we spun the spinner 20 times, we should expect boys to win around 10 times and girls to win around 10 times. He said that 50/50 meant that the boys would win about 50% of the time and that the girls would win about 50% of the time. All of the students agreed that this was a good way to think about fairness for the spinners. Next, we asked the students to think about if all our spinners were fair and what they thought the results might look like for each of our spinners. After the students had pondered this question, we revealed the results on the overhead projector (see Figure 5).

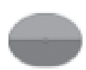
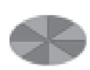
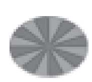
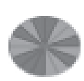
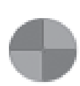
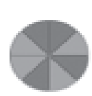
	Girls	Boys
Spinner A 	101 Spins	96 Spins
Spinner B 	89 Spins	91 Spins
Spinner C 	82 Spins	80 Spins
Spinner D 	89 Spins	92 Spins
Spinner E 	98 Spins	94 Spins
Spinner F 	82 Spins	81 Spins

Figure 5. Complied results presented to students.

Many of the students seemed surprised with the results, especially for spinners D and F. After the students viewed the results of 200 spins, we asked them:

How could all of these spinners look different, but the boys and girls won about the same number of times?

The students were asked to jot down ideas about this question for a few minutes, and then the students shared some of their reasons why each of the spinners was different but yielded similar results. Several explanations arose from brainstorming.

Two of the explanations that the students came up with related to the area of the circle. Several of the students seemed to use spinner A as a benchmark spinner to compare the other spinners to. In one of the explanations, the student imagined the sectors of other spinners melting together and becoming Spinner A. In the second explanation, the student imagined breaking the spinners apart by their sectors and rearranging them to make Spinner A. In either case, both students pointed out that the sectors in each of the spinners could be rearranged to represent A or another spinner.

Other students focused on the number of pieces. Some of the students focused on the number of sectors for boys and girls on each spinner, such as comparing the ratios of girl and boy. The students referred to the number of sectors as the number of chances. One student said that the number of chances is equal for each spinner because spinner A has 1 chance for the girls and 1 chance for the boys and spinner B has 4 chances for the girls and 4 chances for the boys. Some students took this further and focused on the size of the sectors. They stated that not only did the students have the same number of chances but the pieces were the same size.

At this point, we decided these were good transitional explanations into fractional equivalence. We asked the students if they had heard of same size pieces before in mathematics. The students responded that they had discussed same size pieces when learning about fractions. We then asked the students:

How can you use fractions to describe the fairness of the spinners mathematically?

The students began by pointing out that in spinner A the chance of winning for a girl or boy was 1 out of 2, in spinner B it was 4 out of 8, and it spinner C it was 8 out of 16, etc… We then asked them to explain further so what would make these the same. How could 1 out of 2 be the same as 2 out of 4 or 8 out of 16? One student said that they are all equivalent fractions. Knowing that this word was not commonplace in the classroom, we asked the students to describe what they thought equivalent meant. Many of them said that it meant that the fractions were the same but looked different. We asked them how they knew they were the same. The students pictorially showed with the spinners that the pieces could be put together to make one another and others began to use symbolic expressions (see Figure 6).

We also asked the students if they could create another spinner that was fair. Students were able to create spinners composed of six pieces as well as ten pieces that were fair and equivalent to the spinners they investigated.

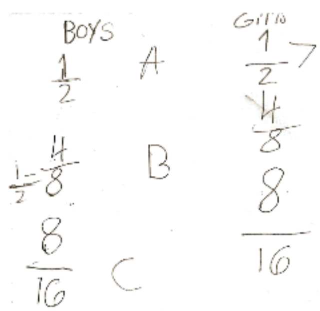


Figure 6. Probability of boys and girls winning.

Lesson Wrap-Up and Reflection

Knowing this was an introductory lesson, we wanted to find out where our students were and what we still needed to address. We asked the students to write a letter to the teacher using the following prompt:

Using your results and the results your classmates found and discussed, please write a note to the teacher telling her which spinner(s) are fair and why.

In many of the letters (See Figures 7 and 8), students discussed cutting, breaking apart, or melting the spinners to show that each of them were the same. Students also discussed the idea of fairness in that both the boys and girls had an equal chance to win.

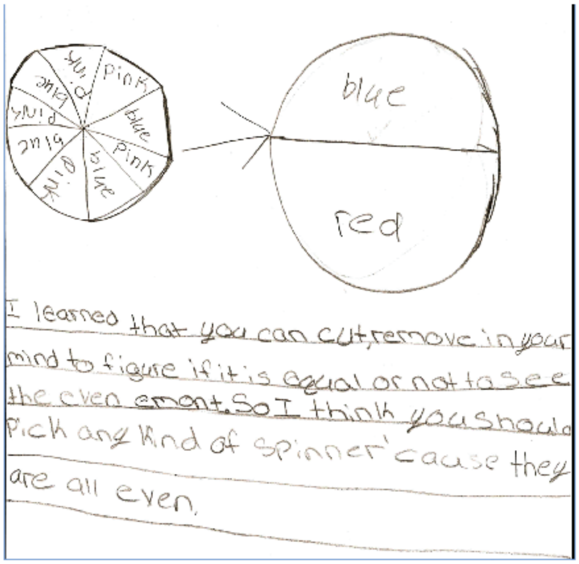


Figure 7. Sample student letter #1.

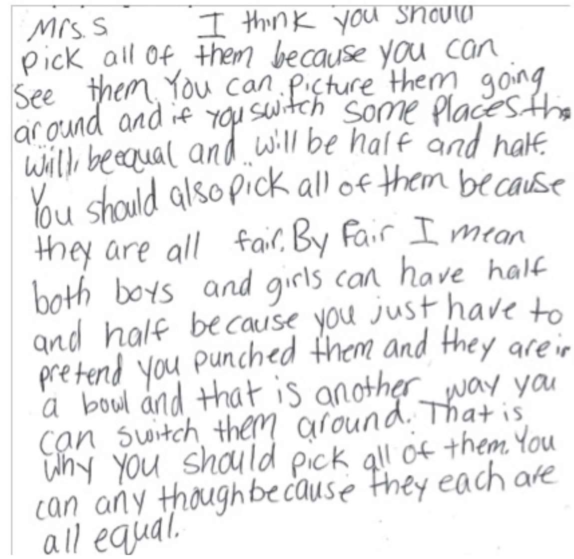


Figure 8. Sample student letter #2.

Conclusion

At the end of the lesson we, as teachers, had several realizations. We initially believed that probability would easily lend itself to the study of rational numbers. Students love to play games and often engage with tools like spinners or dice. As research (Johanning, 2008) indicated, it was not an easy task for our students to apply their rational number reasoning in a new context. The appearance and the arrangement of the spinners swayed their decisions. By allowing the students to interact with the spinners, collect data, and discuss, they were able to use prior rational number reasoning to help explain the phenomenon that they observed.

Probabilistic reasoning and the concept of fairness also allowed students to further define and visualize what it means for fractions to be equivalent. In the fourth grade, according to the Common Core State Standards for Mathematics, students are expected to explain fractional equivalence through visual models. During this activity, students were able to visualize the spinners melting or breaking apart to help further define, for themselves, what it meant for fractions to be equivalent. To further examine students thinking, next time we might ask students to design their own spinners to add to our set and describe why the spinners are fair.

When we integrate different mathematical content domains together, we have to juggle students' misconceptions, superstitions, and understandings within multiple content areas. It often seems easier to focus on one mathematical concept at a time. This lesson highlights that cross-conceptual mathematics lessons are important because they can help extend students' understandings by examining ideas and concepts in new or different ways.

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