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Inquiry, Logic, and Puzzles

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Inquiry, Logic, and Puzzles

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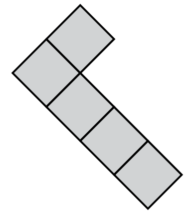
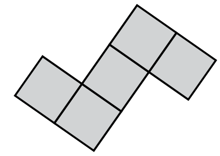




The greatest art doesn't belong to the world. Do not be intimidated by the experts. Trust your instincts. Do not be afraid to go against what you were taught, or what you were told to see or believe. Every person, every set of eyes, has the right to truth.

—Blue Balliett in *Chasing Vermeer*

The first time I read the book *Chasing Vermeer*, I immediately fell in love with the character of Ms. Hussey (Balliett 2005). Ms. Hussey, an elementary teacher at the University of Chicago Laboratory School, challenged her students and entertained whatever problems or questions her students brought to class. Ms. Hussey seemed beyond the epitome of an excellent teacher. One day in class, two of her students, Calder and Petra, informed the class about a recent local burglary of a special Vermeer painting. Through a series of events and suspense, Calder and Petra used inquiry, logic, and puzzles to solve the mystery of the stolen painting.

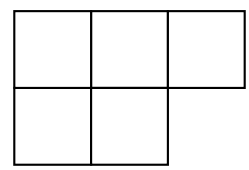
As a secondary mathematics teacher and educator, I am always looking for a good book to implement in a middle school mathematics classroom. *Chasing Vermeer* presented itself as a perfect book to implement in grades 5–8 classrooms. Although *Chasing Vermeer* is not a book explicitly about mathematics, the mathematics and logic that emerged from the book spurred ideas that evolved into tasks for my middle school students to solve (Wessman-Enzinger, Wickstrom 2013). In this article I describe the book and a sequence of activities for grades 5–8.

Mathematics Emerging from *Chasing Vermeer*

The entire book can be thought of as a puzzle. One of the main characters, Calder, used pentomino pieces both as a fun puzzle and as a way to solve the mystery of the missing Vermeer painting. Not only did I love Ms. Hussey, but as a lover of mathematics, I also loved the puzzles. The codes throughout the book, the hidden messages in the illustrations, and the use of pentomino pieces became the themes of many of the tasks we used in my sixth grade class.

Pentomino Pieces

A pentomino is a polyomino constructed from five congruent squares connected by edges. The pentomino pieces are often given letter names based on the similarity between the piece and the letter name. For example, the "P" pentomino looks like the letter P.



Some students may be familiar with polyomino pieces from experiences in playing games. The game Tetris is composed of tetrominoes, which are polyominoes consisting of four congruent squares connected by edges. The game Blokus utilizes polyomino pieces consisting of one to five squares. Despite their presence in *Chasing Vermeer* and games like Tetris and Blokus, many students are unfamiliar with the pentomino pieces.

Task 1: Creating the Pieces

Before my sixth-grade students started reading *Chasing Vermeer*, I introduced the book by showing them the first three polyominoes: the monomino, the domino, and the triomino. After discussing the rules for constructing a polyomino—a polyomino is constructed with congruent squares that are connected to each other by at least one edge—I supplied my students with square-inch foam pieces and graph paper, and I posed the first task: What would a tetromino look like, and how many tetromino pieces are there?

Students constructed as many tetrominoes as they could and then shared them with the class. We noted that some of our pieces looked the same and some looked different. From this, we then discussed the rigid transformational properties of any polyomino and how the pieces may look different but are still the same, with transformations. After constructing and discussing the tetromino, we progressed to the pentomino, using the same questions as before.

Constructing the pentomino pieces seemed more challenging for my class, perhaps because there are significantly more pieces to construct. After constructing the pentomino pieces and discussing the shapes and number of pieces, we displayed the solutions in class. I then gave each student a set of plastic pentominoes to use throughout the *Chasing Vermeer* unit. (Note: the Scholastic website, **www.scholastic.com/titles/chasingvermeer/pentominoes.pdf**, has a printable pentominoes sheet that students can cut out.)

Task 2: Organizing and Looking for Patterns

Constructing all the polyomino pieces for a hexomino, a heptomino, or an octomino would become an arduous task. Much like what Calder and Petra do in the story, I asked my class to organize what they already knew about the number of shapes for the previous polyominoes to predict how many shapes there would be for hexominoes, heptominoes, and octominoes without having to make them. My students eventually decided to organize their data in a table that consisted of three columns (Name of Polyomino, Number of Squares, and Number of Unique Pieces). The table also had a row for each kind of polyomino—from monomino through octomino.

Since the essence of the entire plot of *Chasing Vermeer* is to look for patterns, I challenged the students to look for patterns once the data was organized in a table. Middle school students have used these types of tables mainly for functions that have a linear pattern, and they started looking for linear patterns first and then started looking for recursive relationships. This presented a real challenge for the students since the pattern was not so obvious. Some students resorted to modeling, which is feasible for the hexomino that has 35 unique pieces; however, modeling the heptomino, with 108 unique pieces, is too time-consuming.

Like many puzzles in recreational mathematics, polyominoes present problems that promote the use of combinatorial mathematics, such as determining how many ways these pieces can be arranged. Although mathematicians have estimated and developed algorithms for the estimates to determine how many polyomino pieces there are, no formula has as yet been found that relates the number of squares with the number pieces.

This task promoted organizing data, speculating about our data, and learning that mathematics is a field of unanswered questions. I had given my students an unsolvable problem! Many students never get to experience the wonders of unanswered but still researchable questions in mathematics. By utilizing pentominoes and extending Task 1, this became an outlet for these types of discussions. I even shared the story about Andrew Wiles, who famously solved Fermat's Last Theorem in 1994. He first fell in love with the problem in the 5th grade when his teacher showed him an unsolvable problem. Following in Ms. Hussey's footsteps, I wanted to challenge my students to become the experts and promote autonomy in both creating and solving mathematical tasks.

Task 3: Solving Puzzles

Once or twice a week, as my students read the book, I incorporated tasks that I felt were appropriate with discussion of the book. After my students had created and discussed all the pentomino pieces and encountered an unsolvable problem, I had the pentomino pieces available in the classroom for my students to use to complete puzzles. In addition to the physical puzzles pieces, *Chasing Vermeer* has a website that provides varying levels of digitally dynamic puzzles (**www.scholastic.com/blueballiett/games/index.htm**). Before I presented my class the two versions of the puzzles that I expected them to solve, I had them play the less-challenging puzzles from the website.

Each pentomino piece consists of five square units, and there are a total of 12 pieces. The most challenging puzzles are those that use all 60 square units (12 pieces x 5 square units). I gave my class the task of creating a template for what they thought would be the hardest puzzle and then to explain why. Some puzzles were odd, complicated shapes; some puzzles used all of the pieces; and some templates created by the students were not possible because they did not use areas that were multiples of five.

We discussed all of the templates that were created and conjectured that the most challenging puzzle might be one that not only uses all of the pieces but is also a simple shape, such as a rectangle. Some students argued this point since they thought there would be less "clues" about how to use the puzzle pieces. Because a 12 x 5 rectangle of pentomino pieces has thousands of different solutions, this is a strong argument.

After these discussions, I gave my students two weeks to find two solutions that used all 60 square units of the pentomino pieces. One puzzle they solved was a rectangle that was 12 units by 5 units. The other puzzle was an 8-by-8-unit square with the four corners removed.

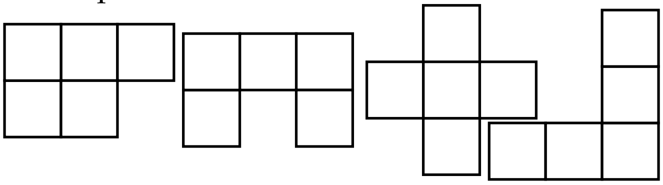
Task 4: Examining Unique Puzzle Solutions

After the students solved the puzzles, they were asked to trace their solutions on a sheet of paper. Since there are many possible ways to solve the puzzles for a rectangle and a square with the corners removed, I shared my expectation to the students that their solutions should be unique. After the students solved and shared their solutions in class, I challenged them to use their current puzzles to generate more solutions.

One way to do this is to look at pairs of pieces. Sometimes when two pieces are reflected, the pieces take up the same amount of area but present a new solution. To demonstrate and explain their mathematics, the students were asked to trace the lines of reflection in their new solution and justify that their transformations did in fact generate new solutions.

Task 5: Perimeter and Area

As the book was read, we discussed how the students in Ms. Hussey's class brought problems and questions they had to class. I challenged my class to come up with a question that they were interested in about pentominoes and bring it to class. The problems posed by the students often dealt with creating shapes of different areas, related to Task 3. To push my students, I wanted to know if they could move beyond simply solving puzzles to thinking about the connections between area and perimeter. For example, I told the students to use four specific pentomino pieces, such as C, X, P, and V, and to find the shape that maximized the perimeter. The students used the pentominoes and modeled the possibilities to help them with the coordination between perimeter and area.



Then I asked the students to determine the specific pieces that would minimize the perimeter for 20 square units. After the students modeled their strategies and solutions, I wanted to facilitate their reflection about strategies to maximize and minimize perimeters and asked them to think about the same problem using 10 or 15 square units without modeling it. The students were pushed to create justifications and counter arguments while thinking about perimeter and area. At the end of the discussions, I prompted the students to model to check their conjectures and arguments.

Conclusion

The book, *Chasing Vermeer*, created the perfect setting for mathematics, embracing puzzle solving, logic, and inquiry. A book does not need to be mathematical for it to serve as a resource in the mathematics classroom (Wessman-Enzinger, Wickstrom 2013). Although the theme of the book did not emphasize number crunching or mathematics, mathematics emerged through the tasks related to the book and with the puzzle pieces that the characters used.

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