2014

Fractions Fall from the Sky

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Recommended Citation

Published in Wisconsin Mathematics Teacher, 65(2), 4–7, 2014.

Fractions Fall from the Sky

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We asked our 5th graders, "If it was raining food outside, what food would you make it rain?" A chorus of excited answers, from "sleeting milkshakes" to "steak and potatoes," echoed throughout the classroom. After reading the book, *Cloudy with a Chance of Meatballs*, together in class and discussing what it would be like if it rained food, our students eagerly entered into a lesson on rational numbers using the context of this book. Fractions fell from the sky as our students engaged in mathematics when it is *Cloudy with a Chance of Meatballs*.

Literature as a Context

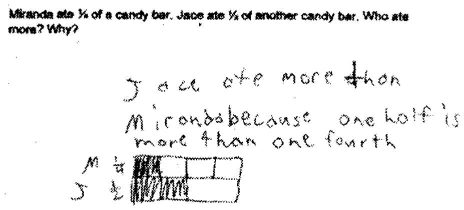
This lesson was implemented twice over a two-year period and was co-taught by the authors to a fifth grade class. Despite the book being below our students' grade level, the use of this non-mathematical picture book was exciting for the children because many of them had also seen the popular movies based on this book. Because children relate to literature and movies on a personal level, building from their experiences in a mathematics lesson can provide a valuable context for students to initiate meaningful mathematical discourse. This type of connection helped us to foster motivation in mathematics and for our lesson (Tucker, Boogan, & Harper, 2010; Whitin & Gary, 1994) This particular book served as a distinctive way to integrate the use of children's literature into our classroom because the book itself is not mathematically-themed (Johanning, Weber, Heidt, Pearce, & Horner, 2009).

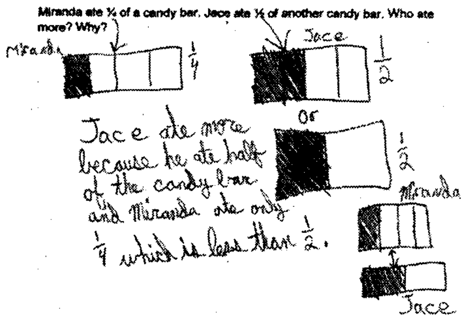
Difficulties with Rational Numbers

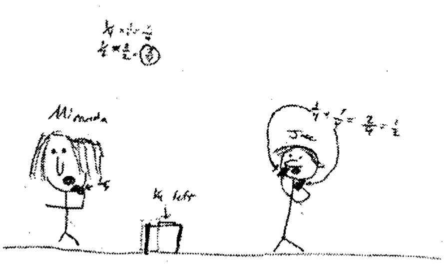
Typical instruction and curricular materials incorporating rational numbers are saturated with fraction bars and circle models. We noticed that within our fifth grade classroom these models were being overused. Research shows that students need exposure to various models that extend beyond the traditional area model when learning about rational numbers to gain flexibility in reasoning and understanding (Petit, Laird, & Marsden, 2010). Research and experience also show that students struggle to re-define the whole, or make sense of the referent unit, when considering rational numbers. Making sense of the referent unit, or a different whole, is an important component of understanding the nature of a rational number because many students treat the multiplicative reasoning of rational numbers as a static entity or a fixed quantity. For example, our students in the fifth grade classroom struggled to distinguish between the *number* ½ and the multiplicative concept of *½ of a quantity*. Conceptually differentiating "½" versus "½ of" is a conceptual struggle for many students. In addition, because the number ½ is always greater than the number ¼, many our students formed the misconception that "½ of" is also always greater than "¼ of."

One day in class, we posed the following problem to our students, "Miranda ate ½ of a candy bar. Jace ate ¼ of another candy bar. Who ate more? Is it possible to know? Explain your reasoning" (Cramer & Whitney, 2011; Petit, Laird, & Marsden, 2010). Every fifth grader, despite over two years of experiences with rational numbers, concluded that Miranda ate more than Jace. From the written work, it seemed that these students were not considering that the whole, or the referent unit, could be different sizes (see Table 1).

Table 1. Typical responses from fifth graders.







It surprised us that none of our students considered the possibility of the wholes being different sizes after instruction. Similarly, we were surprised that our students held strong misconceptions that a half of one candy bar was always more than one fourth of another candy bar, considering their informal experiences of eating various candy bars. If we had only examined the written work, the misconception would appear to be solely rooted with students needing to re-define different wholes; however, upon discussion in class it became apparent that the misconceptions were deeper than just distinguishing different referent units. Our fifth graders voiced, "½ is always bigger than ¼" as if "½ of" and "¼ of" were numbers. Because the number ½ is always bigger than the number ¼ on the real number line, this only re-enforced this deeply rooted misconception and provided them evidence to support their arguments. Our entire class embraced this misconception.

Despite our use of humor and discussion of food, our students still struggled. Furthermore, we noticed that our fifth graders were only using one type of model in their reasoning, a fraction bar, during their daily mathematical interactions and discussion. Even with carefully posed questions, drawing attention to the various representations between the number line, the circle models, and the fraction bars, students would continually share, "½ is always bigger than ¼." These types of discussions led to the creation of our *Cloudy with a Chance of Meatballs*, where we intentionally placed students in positions to mathematically consider and discuss as a class the differences between the *number* ½, and the *multiplicative reasoning of* ½.

The Lesson

Because of the strongly held misconceptions that we observed, one of the goals of our lesson was to promote opportunities to consider other models, such as a discrete-continuous model, rather than just our class's typical go-to fraction bars. Another goal of our lesson was to create a situation that drew students' attention to considering different wholes and to promote reference to their referent unit. We collaboratively created this lesson with the following three objectives:

- Create a meaningful context using children's literature to promote mathematical discourse.
- Utilize a discrete-continuous model to extend the use of different models in our class.
- Facilitate discussion to address the difference between the *number* "½" and the *multiplicative reasoning* of "½ of."

The fifth grade class read the book together and discussed the book before diving into the mathematics. After the context of *Cloudy with a Chance of Meatballs* was established, we created a scenario where the students went outside with a jar and collected meatballs that were falling from the sky. Each group collected a different amount of meatballs. Separated into cooperative learning groups, each student received a jar of fake meatballs (see Figure 1) that simulated the meatballs that they had caught falling from the sky.

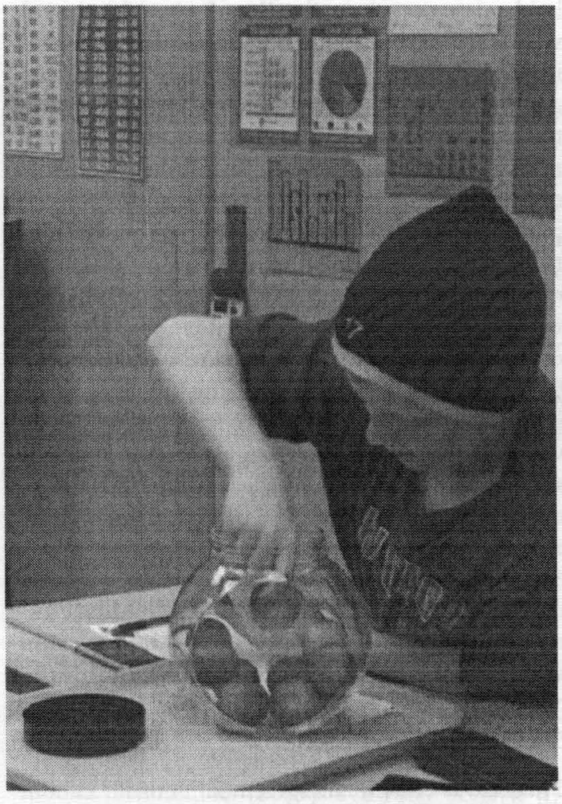


Figure 1. A student working with the meatballs

The meatballs served as our discrete-continuous model, distinct from typical fraction bars or circle graphs. Each jar contained 6, 8, 12, 16, 18 or 20 meatballs. Every group was asked to find ¼, ½, ¾, and 1/3 of the meatballs in their jar. They shared results on a chart that was displayed in front for the whole class to see (see Table 2).

Number of Meatballs	Group #1	Group #2	Group #3	Group #4	Group #5	Group #6
Total Meatballs	6	8	12	16	18	20
$\frac{1}{2}$ of Meatballs	3	4	6	8	9	10
$\frac{1}{4}$ of Meatballs	$1\frac{1}{2}$	2	3	4	$4\frac{1}{2}$	5
$\frac{3}{4}$ of Meatballs	$4\frac{1}{2}$	6	9	12	$13\frac{1}{2}$	15
$\frac{1}{3}$ of Meatballs	2	$2\frac{2}{3}$	4	$5\frac{1}{3}$	6	$6\frac{2}{3}$

Table 2. The different amounts of meatballs given to cooperative groups and multiplicative reasoning generated by the class.

The students were asked to reflect on the results in their groups. After discussion about the results, the student collaboration and classroom discourse centered on the following question: *Do you think there is ever a time when ½ of the meatballs is greater than ¾ of the meatballs? Explain your position and why you think this does or does not happen.*

Building from Our Students' Thinking

The students entered this lesson with the strongly held misconceptions that ½ of something is always smaller than ¾ of something and that "½" is equivalent to "½ of" something. These students were thinking about "½ of" and "¾ of" as the numbers, ½ and ¾, rather than addressing the multiplicative reasoning in "½ of the meatballs" or "¾ of the meatballs." To address this type of multiplicative reasoning, our students also needed to be aware of their referent units. Although the objective of the lesson was centered on what the whole is through examining situations where ½ of the meatballs was larger than ¾ of the meatballs, we did not want to tell the students or directly lead them to that concept. Additionally, we wanted the students to struggle with the language so that conceptions and ideas about the differences between "½" and "½ of" emerged and became a focus we discussed and negotiated (Smith & Stein, 2011). We were intentional about facilitating the discourse in a way to help students discover the need for attention to the referent unit as well as the need to make meaning and differentiate between *the use of number versus multiplicative reasoning.*

There was much disagreement initially about whether ½ of the meatballs could ever be more than ¾ of the meatballs due to their strongly held misconceptions. After discussion and focusing on the chart, some students identified some of the instances in the chart where ½ of the meatballs were larger than ¾ of the meatballs (see Table 3). For example, ½ of the meatballs for Group #2 was 4 meatballs and ¼ of the meatballs for Group #6 was 5 meatballs. Despite some students providing these types of examples, many of the students argued with the entire class that they thought there was a mistake in the counting or recording of the meatballs. These students had not differentiated the meanings of the *numbers* ¼ and ½ versus their *multiplicative reasoning* with "¼ of the meatballs" and "½ of the meatballs." In response to the students' disagreement, we asked the groups to recount their meatballs. When the students recounted and the results stayed the same, some of the students looked confused. We probed the class, "It seems that no one made a mistake. How is this possible? How could one group's '½ of the meatballs' be larger than another group's '¾ of the meatballs'?" Finally, after facilitating discourse and asking more questions, a few students identified and presented to the class the argument, "It's because it's not ½ and ¾. It's ½ of and ¾ of." Discussion then was centered about what the students meant by "of." The students shared that "of" referenced the concept of a different whole and different amounts of meatballs in each group. The students began to identify the importance of the referent unit and identifying the whole, when one takes "½ of" or "¾ of" something. Additionally, students began to grapple with the difficult abstraction of the differences between "½" and "½ of."

Extending the Lesson

The lesson was later extended to other food discussions to incorporate additional fraction models. In addition to the meatballs, each cooperative group was also given pizza boxes with two different sized "pizzas," with one pizza significantly smaller than the other. These paper pizzas, small and large, served as the classic circle models for the students. The students had various tasks with both sized pizzas, which were blank and un-partitioned. For example, by utilizing rulers and protractors the students found 7/8 of both the small

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Total Meatballs	6	8	12	16	18	20
$\frac{1}{2}$ of Meatballs	3	[illegible]	6	8	9	**10**
$\frac{1}{4}$ of Meatballs	$1\frac{1}{2}$	2	3	4	$4\frac{1}{2}$	[illegible]
$\frac{3}{4}$ of Meatballs	$4\frac{1}{2}$	6	**9**	12	$13\frac{1}{2}$	15
$\frac{1}{3}$ of Meatballs	2	$2\frac{2}{3}$	4	$5\frac{1}{3}$	6	$6\frac{2}{3}$

Table 3: The light grey represents a situation where ½ of the meatballs is larger than ¾ of the meatballs. The dark grey represents a situation where ¼ of the meatballs is more than ½ of the meatballs.

and large pizzas. They were asked to consider and explain which pizza was more and why. We also asked the students to find ½ of the small pizza and ¼ of the large pizza. Which was more pizza? Why? The pizzas were intentionally constructed in a way such that ¼ of a large pizza was greater than ½ of a small pizza. Similar to the meatball activity, the pizzas were utilized not only to target the idea of different referent units, but also to counter the students' intuitions that "½ of" is always larger than "¼ of." This extension was an activity to reinforce some of the concepts that were discussed previously with the discrete-continuous model (i.e., meatballs) with their familiar circle model (i.e., the pizzas).

The students did not struggle as much with this activity after their previous discovery and the prior in-depth discussions that took place during the meatball task. At the end of the lessons, students were given problems such as, "Kyle is really hungry. Should he eat 7/8 of a pizza or ½ of another pizza? Explain your recommendation and reasoning." The students' responses on paper were exciting because every student in class referenced that the whole or the size of the pizza matters in the context of their response. This provided some evidence for us that our classroom discussion had made an impact.

Conclusions

A week after implementing our lesson in the fifth grade classes, we asked the students a similar question that targeted consideration of different size wholes. Many of the students drew from their experiences in this lesson and referenced that the whole matters; however, not every student answered this way or made the same connection to this activity. A few students did not refer to the referent unit or applying multiplicative reasoning. This is an illustrative example that learning rational numbers is a difficult pursuit and that these misconceptions are deeply rooted. There was an obvious improvement in the majority of our students' skills in regard to thinking about the whole. Moreover, the activity allowed us to promote exciting mathematical discourse about rational numbers versus multiplicative reasoning in the classroom. We need to provide our students meaningful situations and lessons that they can connect with on a personal level in order to help them understand these challenging mathematical tasks. Utilizing the children's book *Cloudy with a Chance of Meatballs* set the stage for creating mathematical tasks that genuinely interested the students and eventually promoted meaningful mathematical discourse. Although the context of the book was not directly mathematically relevant, we were able to use it successfully in multiple mathematics classrooms to promote discourse about the importance of referencing the whole when discussing rational numbers and distinguishing between "½" and "½ of".

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